MASTER OF COMPUTER APPLICATIONS (MCA)

MCA/ASSIGN/SEMESTER-I

ASSIGNMENTS

(July - 2019 & January - 2020)

MCS-011, MCS-012, MCS-013, MCS-014, MCS-015,

MCSL-016, MCSL-017

SCHOOL OF COMPUTER AND INFORMATION SCIENCES
INDIRA GANDHI NATIONAL OPEN UNIVERSITY
MAIDAN GARHI, NEW DELHI – 110 068

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Important Notes

1. Submit your assignments to the Coordinator of your Study Centre on or before the due date.
2. Assignment submission before due dates is compulsory to become eligible for appearing in corresponding Term End Examinations. For further details, please refer to MCA Programme Guide.
3. To become eligible for appearing the Term End Practical Examination for the lab courses, it is essential to fulfill the minimum attendance requirements as well as submission of assignments (on or before the due date). For further details, please refer to the MCA Programme Guide.
4. The viva voce is compulsory for the assignments. For any course, if a student submitted the assignment and not attended the viva-voce, then the assignment is treated as not successfully completed and would be marked as ZERO.

Course Code	**:**	**MCS-013**
Course Title	**:**	**Discrete Mathematics**
Assignment Number	**:**	**MCA(I)/013/Assignment/2019-20**
Assignment Marks	**:**	**100**
Weightage	**:**	**25%**
Last Dates for Submission	**:**	**15th October, 2019 (For July, 2019 Session)** **15th April, 2020 (For January, 2020 Session)**

There are eight questions in this assignment, which carries 80 marks. Rest 20 marks are for viva-voce. Answer all the questions. You may use illustrations and diagrams to enhance the explanations. Please go through the guidelines regarding assignments given in the Programme Guide for the format of presentation.

Q1.

(a) What is proposition? Explain different logical connectives used in proposition with the help of example. **(3 Marks)**

(b) Make truth table for followings. **(4 Marks)**

i) $p \rightarrow (q \vee r) \wedge p \wedge \sim q$

ii) $p \rightarrow (\sim r \vee \sim q) \wedge (p \vee r)$

(c) Give geometric representation for followings: **(3 Marks)**

i) R x { 2}

ii) {1, 5) x (-2, -3)

Q2.

(a) Draw a Venn diagram to represent followings: **(3 Marks)**

i) $(A \cap B \cup C) \sim A$

ii) $(A \cup B \cup C) \cap (B \cap C)$

(b) Write down suitable mathematical statement that can be represented by the following symbolic properties. **(4 Marks)**

i) $(\exists x) (\exists z) (\forall y) P$

ii) $\forall (x) (\forall y) (\exists z) P$

(c) Show whether $\sqrt{3}$ is rational or irrational. **(3 Marks)**

Q3.

(a) Explain inclusion-exclusion principle with example. **(2 Marks)**

(b) Make logic circuit for the following Boolean expressions: **(4 Marks)**

i) (x ' y ' z) + (x ' y ' z)'

ii) (x ' yz) (x ' yz ') (xy ' z)

(c) What is a tautology? If P and Q are statements, show whether the statement $(P \rightarrow Q) \vee (Q \rightarrow P)$ is a tautology or not. **(4 Marks)**

Q4.

(a) How many words can be formed using letter of STUDENT using each letter at most once?

i) If each letter must be used,

ii) If some or all the letters may be omitted. **(2 Marks)**

(b) Show that: **(2 Marks)**

P=>Q and $(\sim P \vee Q)$ are equivalent.

(c) Prove that n! (n + 2) = n!+ (n +1)! **(4 Marks)**

(d) Explain principal of duality with the help of example. **(2 Marks)**

Q5.

(a) How many different professionals committees of 10 people can be formed, each containing at least 2 Professors, at least 3 Managers and 3 ICT Experts from list of 10 Professors, 6 Managers and 8 ICT Experts? **(4 Marks)**

(b) What are Demorgan's Law? Explain the use of Demorgen's law with example. **(4 Marks)**

(c) Explain addition theorem in probability. **(2 Marks)**

Q6.

(a) How many ways are there to distribute 17 district objects into 7 distinct boxes with:

i) At least two empty box.

ii) No empty box. **(3 Marks)**

(b) Explain principle of multiplication with an example. **(3 Marks)**

(c) Set A,B and C are: A = {1, 2, 5,7,8,9,12,15,17}, B = { 1,2, 3 ,4, 5,9,10 } and C { 2, 5,7,9,10,11, 13}.
Find $A \cap B \cup C$, $A \cup B \cup C$, $A \cup B \cap C$ and (B~C) **(4 Marks)**

Q7.

(a) Find how many 3 digit numbers are even? **(2 Marks)**

(b) What is counterexample? Explain how counter example helps in problem solving. **(3 Marks)**

(c) What is a function? Explain following types of functions with example. **(3 Marks)**

i) Surgective

ii) Injective

iii) Bijective

(d) Write the following statements in symbolic form: **(2 Marks)**

i) Mohan is poor but happy

ii) Either work hard or be ready for poor result

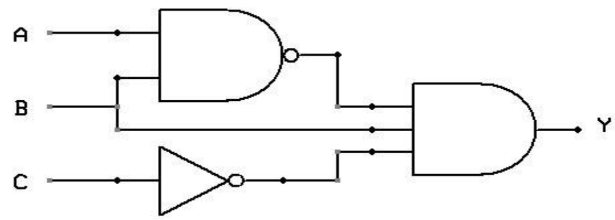
Q8.

(a) Find inverse of the following function: **(2 Marks)**

$$f(x) = \frac{x^2 + 6}{x - 2} \qquad x \neq 2$$

(b) What is relation? Explain equivalence relation with the help of an example. **(3 Marks)**

(c) Find dual of Boolean Expression for the output of the following logic circuit. **(3 Marks)**



(d) Explain distributive and complement properties of set with the help of examples.

(2 Marks)

Session 2019-20

Course Code : MCS-013 Course

Title : Discrete Mathematics

Q1.

(a) What is the proposition? Explain different logical connectives used in propositions with the help for example.

Ans:

It is a declarative sentence, that is either true or false, but not both. Proposition should be either uniformly true or uniformly false.

Example of some proposition are:

Two plus two equals four.

Two plus two equals five.

x + y > 0 for x > 0 and y > 0.

Logical connectives:

These are words, phrases, connectives or links used to combine two or more sentences.

There are four types of logical connectors.

Disjunction: The disjunction of two propositions p and q is the compound statement p or q, denoted by p v q.

Truth table:

p	q	p v q
T	T	T
T	F	T
F	T	T
F	F	F

⇒ **Let p be statement "Arun is an Engineer" and q is "Puneet is teacher" The p v q is given by, Arun is an Engineer or puneet is a teacher.**

Conjection

we call the compound statement "p and q" the conjunction of the statement p and q .
It is denoted this by p ^ q.

[p and q]   [p ^ q]

<u>Truth table:</u>

p	q	p ^ q
T	T	T
T	F	F
F	T	F
F	F	F

True only when both p,q are true

⇒ **Find the conjunction of the proposition p and q where p is "Today is monday and it is raining today".**
The statement is, Today is Monday and it is raining today.

Negation

The negation of a proposition p is 'not p' denoted by ~p.

<u>Truth table</u>:

p	~p
T	F
F	T

<u>Ex</u>: p - Neha is a good girl.
~p - Neha is not a good girl.

Conditional connection: A statement of the form p -> q
Is called a conditional statement or conditional proposition.
We denote the statement 'if p, then q' by p->q we read this 'p implies q' or 'P is sufficient q' or 'p only if q. P is called consequent (conclusion).

<u>Truth table:</u>

p	q	p →q
T	T	T
T	F	F
F	T	T
F	F	T

"If it is Friday then it is raining today" is a proposition which is of the form p -> q. The above proposition is true if it is not Friday(premise is false) or if it is Friday and it is raining, and it is false when it is Friday but it is not raining.

Biconditional: Let p and q be two propositions. The compound statement (p → q) ∧(q → p) is the biconditional of p and q. We denote it by p ↔ q, and read it as 'p if and only q'.
We also say that 'p implies and is implied by q'. or 'p is necessary and sufficient for q'. Truth table:

p	q	q → q	q → p	P ↔ q
T	T	T	T	T
T	F	F	T	F
F	T	T	F	F
F	F	T	T	T

(b) Make a truth table for the following.

i) $p \rightarrow (q \vee r) \wedge p \wedge \sim q$

ii) $p \rightarrow (\sim r \vee \sim q) \wedge (p \vee r)$

Ans:

1.) Truth table:

p	q	r	(q v r)	~ q	(q v r)^p	(q v r)^p^~q	p→ (q v r)^p^~q
T	T	T	T	F	T	F	F
T	T	F	T	F	T	F	F
T	F	T	T	T	T	T	T
T	F	F	F	T	F	F	F
F	T	T	T	F	F	F	T
F	T	F	T	F	F	F	T
F	F	T	T	T	F	F	T
F	F	F	F	T	F	F	T

Truth table:

p	q	r	~r	~q	~r v ~q	p v r	~r v ~q ^ p v r	p→>~r v ~q ^ p v r
T	T	T	F	F	F	T	F	F
T	T	F	T	F	T	T	T	T
T	F	T	F	T	T	T	T	T
T	F	F	T	T	T	T	T	T
F	T	T	F	F	F	T	F	T
F	T	F	T	F	T	F	F	T
F	F	T	F	T	T	T	T	T
F	F	F	T	T	T	F	F	T

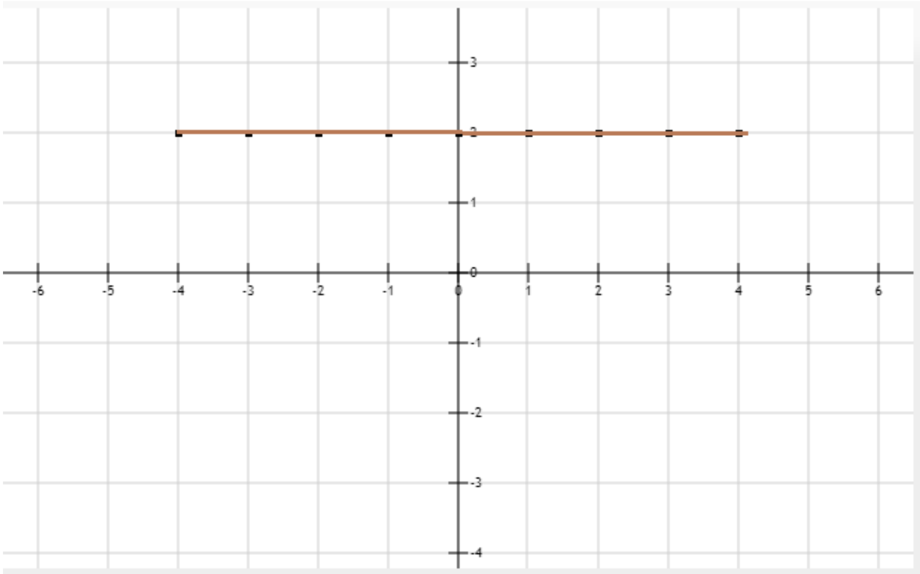
(c) Give geometric representation for followings:

i) R x {2}

ii) (1, 5) x (-2, -3)

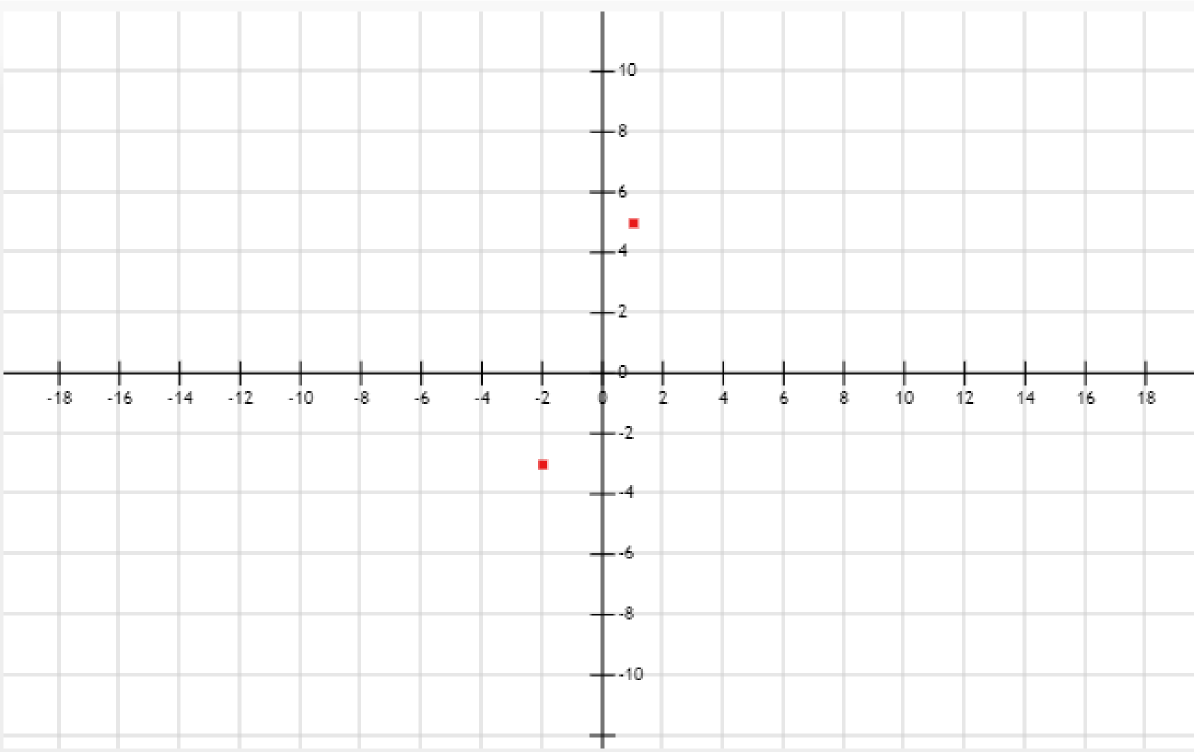
Ans:

i) Rx{2}



ii) (1,5) x (-2, -3)

Ans:



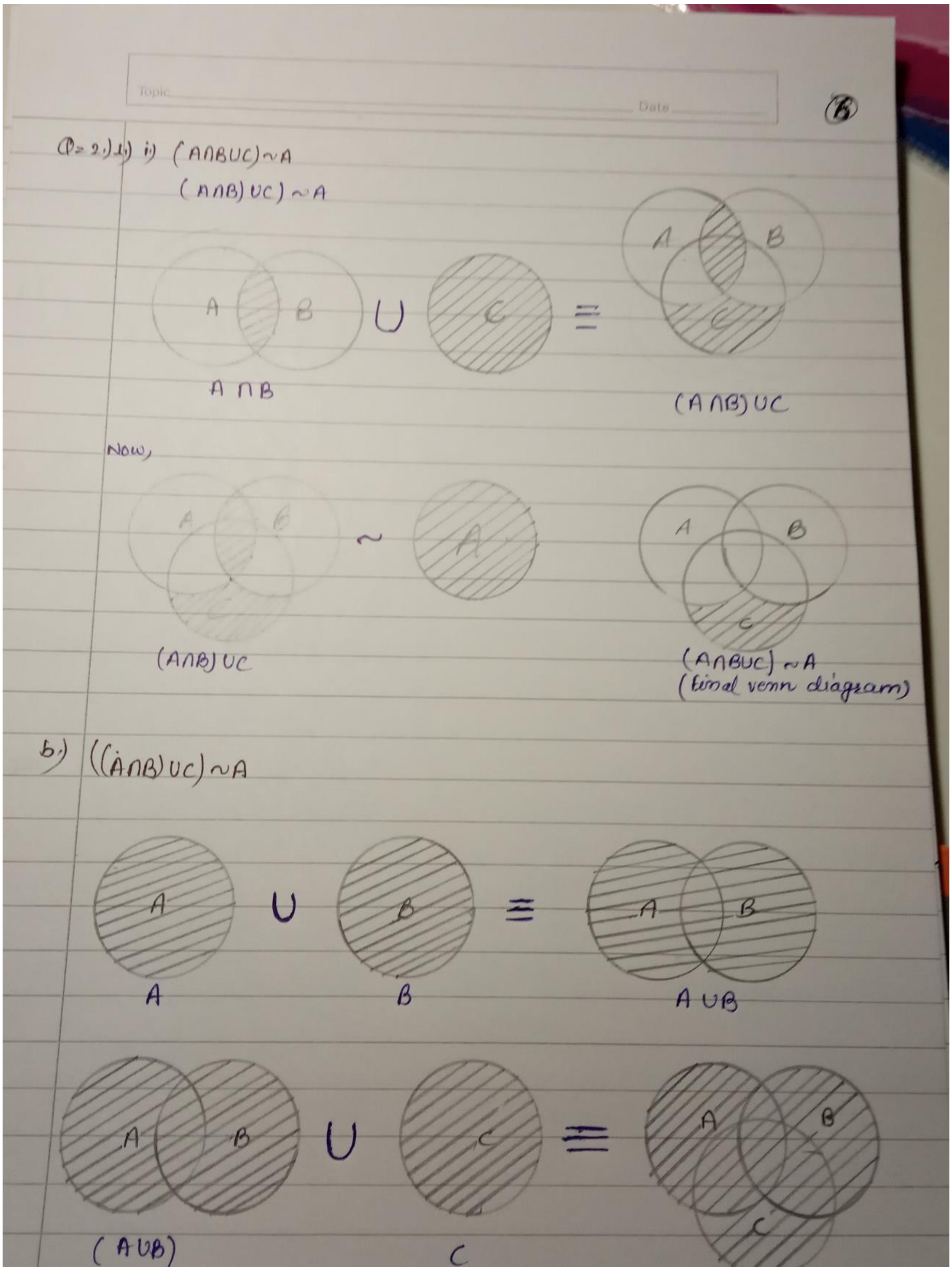
Q2.

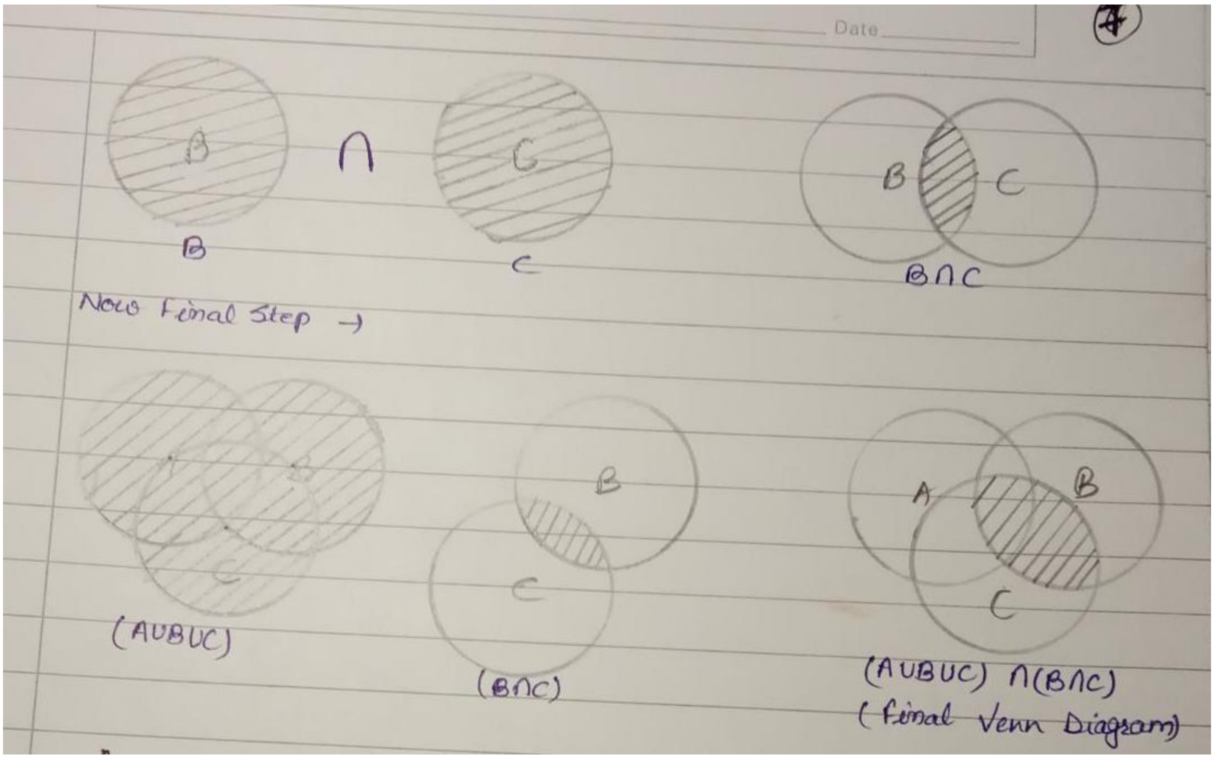
(a) Draw a Venn diagram to represent followings: (3)

i) (A ∩ BU C) ~A

ii) (A U B U C) ∩ (B ∩ C)

Ans:





(b) Write down suitable mathematical statement that can be represented by the following symbolic properties.

i) $(\exists x)(\exists z)(\forall y)\, P$

ii) $\forall (x)(\forall y)(\exists z)\, P$

Ans:

1) **There exist some value x and z and all values of y belongs to Function p.**
2) **All x, some y and some z belong to the function p.**

(c) Show whether √3 is rational or irrational.

Ans. **Let's assume √3 is a rational number. Then we can write it as**

= √3 = a/b (where a,b E whole, b≠0 and a&b have no common factor)

From the equality,√3 = a/b

3 = a2/b2

a2 = 3b2 (it is divisible by 3)

Now consider, a = 3n

∴ 3= a2/b2

3 = (3n2)/ b2

3= 9n2/b2

b2= 9n2/3

b= 3n2

It means that b2 is also a multiple of 3 from which follows that b itself is a multiple of 3.

Hence, our assumption (i.e., √3 is a rational number) contradicts.

∴ It is proved that √3 is a rational number.

Q3.

(a) Explain inclusion-exclusion principle with example.

Ans. **Inclusion-Exclusion principle is a counting technique that computes the number of elements that satisfy at least one of the several properties while guaranteeing that elements satisfying more than one property are not counted twice.**

Example- How many numbers from 0 to 999 are not divisible by either 5 or 7? Ans.. let the objects be the integers 0,1,. .... ,999.

Let A1 be the set of numbers divisible by 5, and A2 the set of numbers divisible by 7.

Now, N=1000 , (A1)=200, (A2)=143

(A1∩A2) = 29

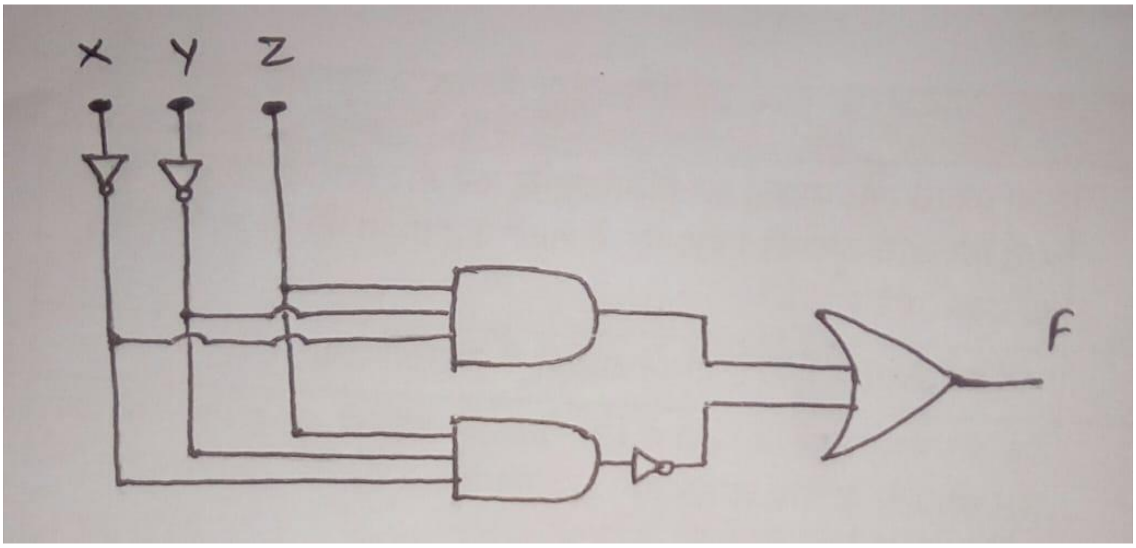
So, by theorem, the answer is 1000 200-143+29

= 686

(b) Make logic circuit for the following Boolean expressions: i)

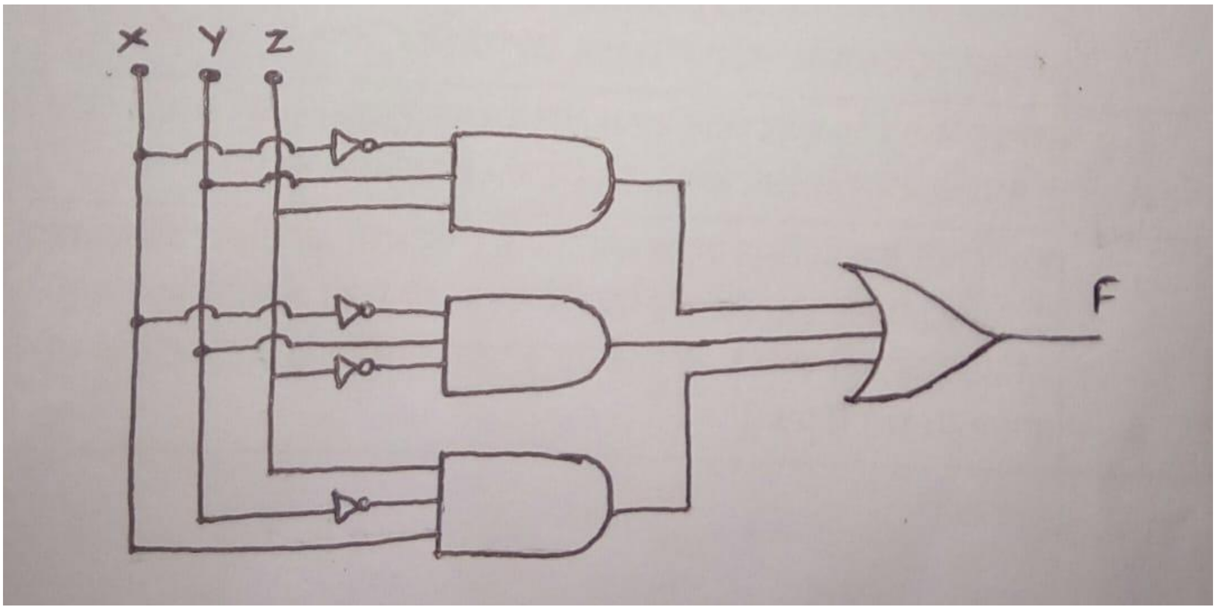
(x'y'z) + (x'y'z)'

Ans:



ii) (x'yz)+ (x'yz')+ (xy'z)

Ans:



(c) What is a tautology? If P and Q are statements, show whether the statement $(P \rightarrow Q) \vee (Q \rightarrow P)$ is a tautology or not.

Ans.

A tautology is a formula which is always True:- that is, it is true for every assignment of truth values to its simple values. A tautology can be think as a Rule of Logic.

(p→q) V (q→p)

Construct the truth table for (p→q) v (q→p) and show that formula is always true.

P	Q	p→q	q→p	(p→q) v (q→P)
T	T	T	T	T
T	F	F	T	T
F	T	T	F	T
F	F	T	T	T

The last column contains only T's. Therefore the formula is a tautology.

Q4.

(a) How many words can be formed using a letter of STUDENT using each letter at most once?

i) If each letter must be used,

Ans. "STUDENT " has 7 letters, all different.

That "7 position 7" or '7P7' = 7*6*5*4*3*2*1 = 5040

ii) If some or all the letters may be omitted.

Ans. That's 7P0 + 7P1 +7P2 + 7P3 +7P4 + 7P5 + 7P6 +7P7 1+7+42+210 + 840 + 2520 + 5040 + 5040 = 13699

(b) Show that:

P=>Q and (~P Q) are equivalent.

Ans. truth table:

p	q	~p	p→q	~p v q
F	F	T	T	T
F	T	T	T	T
T	F	F	F	F
T	T	F	T	T

From the truth table, we have seen that the output of (P→Q) & (∼P v Q) is same. Hence, P→Q & (∼P v Q) are equivalent.

(c) Prove that n! (n + 2) = n!+ (n +1)!

Ans. Proof: n!+ (n+1)! =

n! + (n+1)* (n)(n-1)(n-2) ---------------------- =

n! + (n+1)n! =

n!+ (1+(n+1)=

n!(n+2)

OR

n!(n+2)=

n![(n+1)+1]=

n!*(n+1)+1*n!=

(n+1)!+n! = n!+(n+1)!

Hence proved

(d) Explain principle of duality with the help of example.

Ans. According to Principle of duality, " Dual of one expression is obtained by replacing AND(.) with OR(+) and OR with AND together with replacement of 1 with 0 and 0 with 1.

For Ex: Consider the expression A+B =0. The dual of their expression is obtained by replacing + with . and 0 with 1.

I.e. A.B =1 is dual of A+B = 0

Q5.

(a) How many different professionals committees of 10 people can be formed, each containing

at least 2 Professors, at least 3 Managers and 3 ICT Experts from list of 10 Professors, 6 Managers and 8 ICT Experts?

Ans.

The 10 member of community of professionals can be either of the following- 2 Professors, at least 3 Managers and 3 ICT Exports

Total number of ways: 10c2 x 6c3 x 8c3

= 10 x 9/ 2x1 X 6x5x4/ 3x2x1 X 8x7x6/ 3x2x1

= 90/2 X 120/6 X 336/6

= 45 X 20 X 56

= 50400 ways

So, there are 50400 ways.

(b) What are Demorgan's Law? Explain the use of Demorgen's law with example.

Ans.

De Morgan's laws are a pair of transformation rules that are both valid rules of inference.

These rules are

The complement of the union of two sets is the same as the intersection of their complements; and the complement of the intersection of two sets is the same as the union of their complements.

OR

not(A or B) = not A and not B not(A and B)= not A or not B

For Ex. Let U= {1,2,3,4,5,6}, A= {2,3}, and B={3,4,5}. Show that (AUB)'=a'∩ B'

Sol. AUB= {2,3} U {3,4,5} = {2,3,4,5}

∴ (AUB)’ = {1,6}

Also,

A’= {1,4,5,6} , B’ = {1,26}

∴ A’∩ B’ = {1,4,5,6} ∩ { 1,2,6}

{1,6}

Hence, (A U B)’ =A’ ∩ B’

The use of De-Morgan’s laws:

1) **It is used to simplify Boolean expressions to build equations using only one sort of gate, NAND or NOR gates.**
2) **These laws are helpful in making valid inferences in proofs and deductive arguments.**

(c) Explain addition theorem in probability.

Ans. The Addition Theorem in probability is the process of determining probability that one or more events occur.

Theorem 1. For any two events A and B, the probability that either event ‘A’ or event ‘B’ occurs or both occurs is

$$P(A\cup B) = P(A) + P(B) - P(A\cap B)$$

Theorem 2. For any three events A,B and C, the probability that any one of the events occurs or any two of the events occur or all the three events occur is,

$$P(A\cup B\cup C) = P(A) + P(B) + P(C) - P(A\cap B) - P(B\cap C) - P(A\cap C) + P(A\cap B\cap C)$$

Theorem 3. For any two mutually exclusive events A and B, the probability that either A or B occurs is given by the sum of individual probabilities of A and B,

$$P(A\cup B) = P(A) + P(B$$

Theorem 4. For any three mutually exclusive events A,B and C the probability that the event A or B or C occurs is given by the sum of individual probabilities of A,B and C.

$$P(A\cup B\cup C) = P(A) + P(B) + P(C)$$

Q6.

(a) How many ways are there to distribute 17 district objects into 7 distinct boxes with:

i) At least two empty box.

ii) No empty box.

Ans:

At least two empty boxes =21 * 5^{17}no

Empty boxes = 24 * 5^{13}

Step-by-step explanation:

17 distinct objects into 7 distinct boxes

(i) At least two empty box

First select two empty boxes from 7 boxes

$^7P_2$ = 21

Now 17 distinct object in 5 distinct boxes Each

object can be placed in 5 ways

so 17 objects can be placed in 5^{17} ways Total ways

= 21 * 5^{17}

(ii) No empty box

First take 5 distinct objects and put in 5 distinct boxes

= 5 ! ways

= 120

now we remained with 12 Distinct objects & 5 Distinct boxes Each

object can be placed in 5 ways

so 17 objects can be placed in 5^{12} ways Total

ways = 120 * 5^{12}

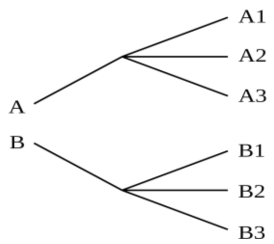
= 24 * 5^{13}

(b) Explain principle of multiplication with an example.

Ans:

In combinatorics, the rule of product or multiplication principle is a basic counting principle (a.k.a. the fundamental principle of counting). Stated simply, it is the idea that if there are a ways of doing something and b ways of doing another thing, then there are a . b ways of performing both actions.

EXAMPLE:



(c) Set A,B and C are: A = {1, 2, 5,7,8,9,12,15,17}, B = { 1,2, 3 ,4, 5,9,10 } and C { 2, 5,7,9,10,11, 13}. Find A∩B∪C, A∪B∪C, A∪B∩C and (B~C)

Ans. 1) A∩B∪C = {1,2,5,7,8,9,10,11,13}

2) A∪B∪C = {1,2,3,4,5,7,8,9,10,11,12,13,15,17}

3) A∪B∩C= {1,2,4,5,7,8,9,10}

4) (B ~ C) = B -C = {1,3,4}

Q7.

(a) Find how many 3 digit numbers are even?

Ans.

3 digits numbers means we have to put 0-9 digit at 3 places, ------ ABC,

At the position of A(First Place) we can put any digits 1-9 not 0, otherwise it will not a 3 digit number, so by 9 ways

At the position B (second place) we can put 0-9 i.e. by 10 ways

At the position of C (third place), we can put 0,2,4,6 or 8 i.e by 5 ways

So total no of digits = 9*10*5

= 450

Total no of three digits even no = 450

(b) What is a counter example? Explain how a counter example helps in problem solving.

Ans. Counter Example: IT is a particular case which displays that a general declaration is false. It is an example with a negative connotation, whereas an example may be used to support or illustrate a claim, a counter example is used to refuse an assertion. Counter example is used to solve problems: Counterexamples are often used to prove the limitations of possible theorems. By using counter examples we display that definite estimations are false, mathematical researchers avoid going down blind paths and learn how to modify estimations to produce demonstrable theorems.

(c) What is a function? Explain the following types of functions with example.

i) Surjective
ii) Injective
iii) Bijective

Ans:

A function f from A to B is an assignment of exactly one element of B to each element of A (A and B are non-empty sets). A is called Domain of f and B is called co-domain of f. If b is the unique element of B assigned by the function f to the element a of A, it is written as f(a) = b. f maps A to B. means f is a function from A to B,

it is written as $f: A \to B$

Terms related to functions:

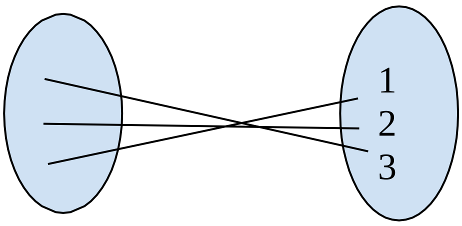
- **Domain and co-domain – if f is a function from set A to set B, then A is called Domain and B is called co-domain.**
- **Range – Range of f is the set of all images of elements of A. Basically Range is subset of co- domain.**
- **Image and Preimage – b is the image of a and a is the preimage of b if f(a) = b.**

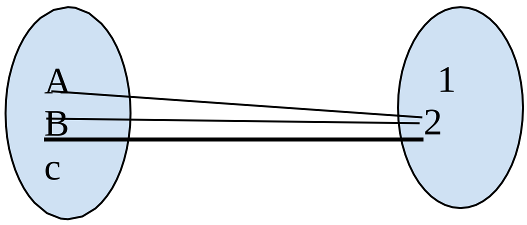
Types of functions

1) **Injective :** **A function f is one-to-one or injective, if and only if f(x)=f(y) implies x=y for all x and y in the domain of f.**

   "All elements in the domains of f have different images."

   F: A→B is injective ⇔ $\forall x_1 x_2$ E A ($f(x_1) = f(x_2) \rightarrow x_1 = x_2$) Ex.

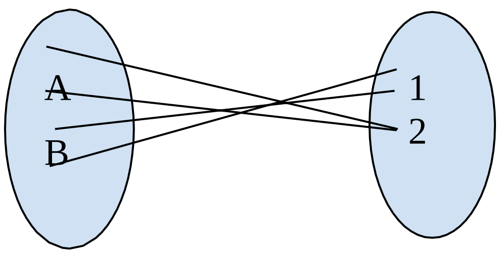


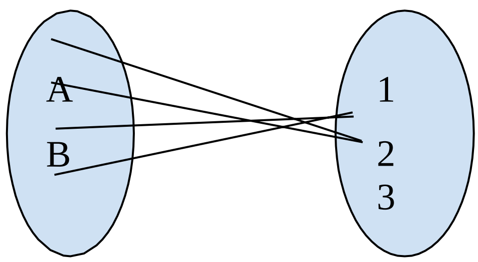


All

2) **Surjective:** **A function f from x to y is onto or surjective if and only if for every element YEy there is an element XEx = y**

   " Each element in the co-domain of f has a pre-image" F:x→y is onto ⇔ ∀y ∃x,f(x) =y





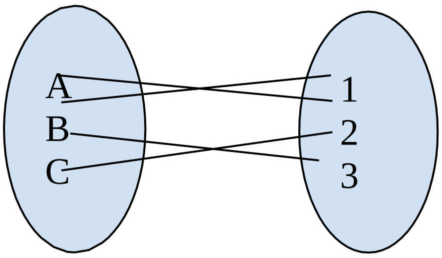
Not

All elements

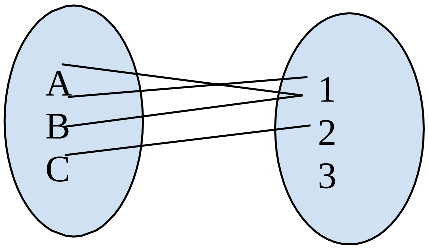
1 has no

3) **Bijective:** **A function f is a one-to-one corresponding of bijective if and only if it is a both one-to-one and onto.**

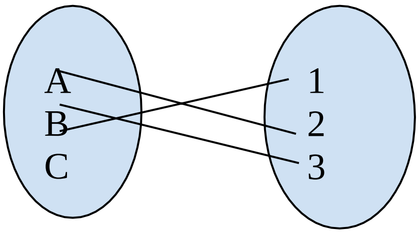
“ No element in the co-domain of f has two or more preimages.”(one-to-one)
and
“ Each element in the co-domain of f has a preimage”.(onto) Ex:



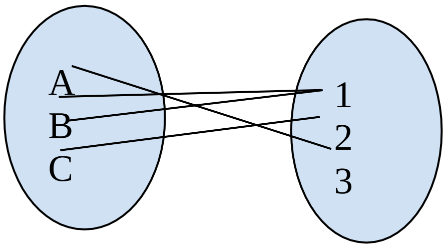
Each element has



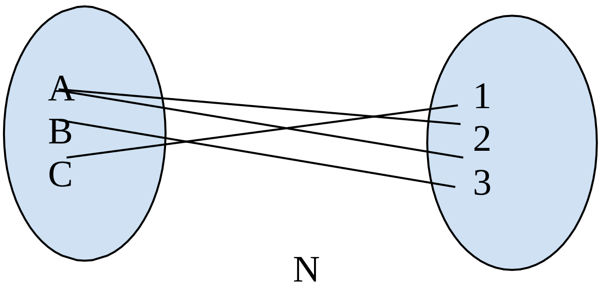
Neither one-to-



No on-to,2



Not one-to-one,



N

Not a

(d) Write the following statements in symbolic form:

i) Mohan is poor but happy

Ans: Mohan = A∧B, where A = Set of poor, B= Set of happy (here “but” translate into “AND”.

ii) Either work hard or be ready for poor result.

Ans: Symbolic Form: $A \wedge \sim B$, where A = set of hardworker
B= Set of good result
$\sim B$ = Negation of B, i.e Set of poor result

Q8.

(a) Find inverse of the following function:

Ans:

$$x = \frac{-(-y) \pm \sqrt{(y)^2 - 4(2y+6)}}{2 \cdot 1}$$

$$= \frac{y \pm \sqrt{y^2 - 8y - 2y}}{2}$$

Therefore, inverse of (x) i.e

$$f^{-1} = \frac{y \pm \sqrt{y^2 - 8y - 2y}}{2}$$

(b) What is relation? Explain equivalence relation with the help of an example.

Ans. Relations: A relation between two sets A and B is a subset of AxB. Any subset of AxA is a relation on the set A.

Ex: if A= {1,2,3}, B ={p,q}, then subset of {(1,0),(2,q),(2,p)}
Is a relation on AxB. And {(1,1),(2,3)} is a relation on A.

Equivalence Relations: A relation R on a set A is called an equivalence relation if and only if

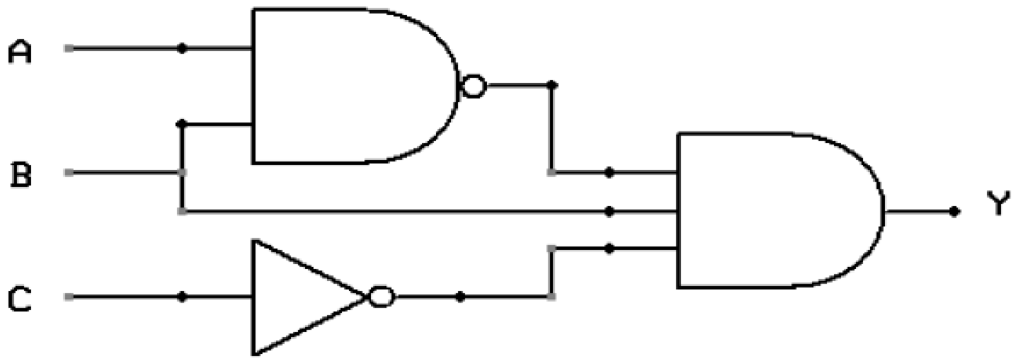
1) **R is reflexive,i.e, for all aER ,(a,a)E R**
2) **R is symmetric, i.e, (a,b) E R ⇒ (b,a) E R , for all a,b E A and**
3) **R is transitive, i.e, (a,b) E R and (b,c) E R ⇒ (a,c)E R, for all a,b,c E A.**

For Example: The relation "is equal to", denoted "=", is an equivalence relation on the set of real numbers since x,y,z E R

1) **(Reflexivity) x=x**
2) **(Symmetry) if x = y, & y=x**
3) **(Transitivity) if x = y, y = z then x = z**

All these are true.

(c) Find dual of Boolean Expression for the output of the following logic circuit.



Ans:
The given Boolean Logic Circuit,

$Y = (A \cdot B)'C'$

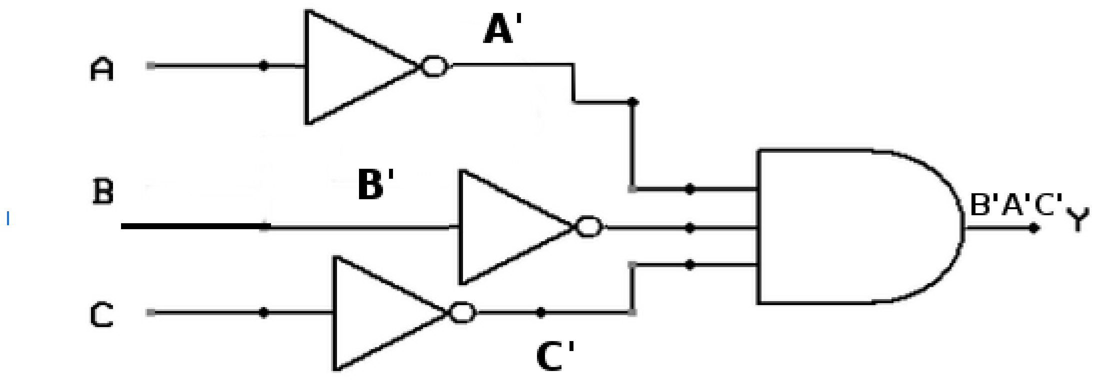
⇒**Dual of** $(A \cdot B)'C'$ **is given below,**

$(A \cdot B)'C' = (B' \cdot A')'$ **(According to De Morgan's Law**

$= B'A'C'$ $(A \cdot B)' = (B' \cdot A')$**)**

Logic circuit diagram is

Given below



$(A \cdot B)'C' = B'A'C'$ **(Hence proved!)**

(d) Explain distributive and complement properties of set with the help of examples.

Ans.

Distributive property of set: This law states that, the sum and product remain the same value even when the order of the elements is altered.

1st Law: It states that taking union of a set to the intersection of two other sets is the same as taking the union of the sets is the same as taking the union of the original set and both the other two sets separately and then taking the intersection of the result.

AU(B∩C) = (A UB)∩(AUC)

2nd Law: It states that taking the intersection of a set to the union of two other sets is the same as taking the intersection of the original set and both the other sets separately and then taking the union of the result.

A∩(BUC) = (A∩B)U(A∩C)

Complement property of sets: In set theory, the complement of a set A refers to elements not in A. When all the considerations are considered to be subsets of a given set U, the absolute complement of a set A is the set of elements in U but not in A.

The relative complement of A with respect to a set B, also termed the difference of sets A and B, written B/A, is the set of elements in B but not in A.

For Example: U = {X:X is an integer) and A ={X:X is an even integer}, then find AC?
Here, U = {X:X is an integer}
A= { X:X is an even integers)
Now, AC = U-A
= {X:X is an integer} - {X:X is an even integer}
Therefore, AC = {X:X is an odd integer}